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Full component Lagrangian in the linear multiplet formulation of string-inspired effective supergravity

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Abstract
We compute the component field four-dimensional $N = 1$ supergravity Lagrangian that is obtained from a superfield Lagrangian in the $U(1)_K$ formalism with a linear dilaton multiplet. All fermionic terms are presented. In a variety of important ways, our results generalize those that have been reported previously, and are flexible enough to accommodate many situations of phenomenological interest in string-inspired effective supergravity, especially models based on orbifold compactifications of the weakly coupled heterotic string. We provide for an effective theory of hidden gaugino and matter condensation. We include supersymmetric Green–Schwarz counterterms associated with the cancellation of $U(1)$ and modular duality anomalies; the modular duality counterterm is of a rather general form. Our assumed form for the dilaton Kähler potential is quite general and can accommodate Kähler stabilization methods. We note possible applications of our results. We also discuss the usefulness of the linear dilaton formulation as a complement to the chiral dilaton approach.

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1. Introduction

The topic of *four-dimensional* $N = 1$ *supergravity* coupled to supersymmetric matter and super-Yang–Mills fields is an old and well-understood part of supersymmetric field theory. A variety of superspace methods were developed many years ago, all of them designed to write locally supersymmetric Lagrangians in a compact form while at the same time leading most easily to component field expressions. Nevertheless, this *component expansion* can be tedious and it proves useful to have the results tabulated; e.g., so that model-building can proceed, with a minimum of effort, from superfield assumptions to component field consequences.

For the case of chiral multiplets coupled to supergravity and the vector supermultiplets of a super-Yang–Mills theory, component expansions have been tabulated under a very broad set of assumptions (originally by Cremmer *et al* [1]; nicely reviewed by Wess and Bagger [2]). For less conventional arrangements and assortments of $N = 1$ multiplets, however, the coverage is a bit patchy. The example in which we are interested involves a *linear multiplet* [3]. Although supergravity coupled to a linear multiplet [4] has been studied in a large number of works (enumerated and discussed below), some details remain to be given. This is particularly true when very general assumptions for the form of the superfield Lagrangian are envisioned.

In the present work we consider a class of supergravity theories—containing a linear multiplet—for which only specialized or somewhat incomplete results are available; it is our intention to generalize previous work and to fill in some details that are missing in the literature.

While completeness is a reasonable motivation for the determination of the full component Lagrangian, Planck mass suppressed fermion interactions may have phenomenological applications. For example, processes forbidden in the *standard model* and globally supersymmetric extensions might be mediated by gravitationally suppressed interactions. This is particularly true in the case where large vacuum expectation values (vevs) occur due to the presence of an anomalous $U(1)$.

Below, we will argue that a theory of supergravity that contains a linear multiplet is well motivated from the perspective of *string-inspired effective supergravity.* Our discussion is a synopsis of opinions offered previously by other authors. Whereas the *chiral dilaton* formulation is more common, on the basis of the points raised in our discussion it is our opinion that the *linear dilaton* framework should be regarded as a useful complement.

We are interested in generalizations of the effective theory of Binétruy, Gaillard and Wu (BGW) [5–7], as well as in the computation of those fermionic terms in the Lagrangian that were neglected by these authors (justifiably, as the issues in which they were interested did not require knowledge of these terms). The *BGW effective theory* is inspired by orbifold compactifications of the weakly coupled heterotic string[1]. The low energy limit of the heterotic string compactified on an orbifold is an effective supergravity theory. The BGW effective theory is designed to implement dynamical supersymmetry breaking through the strong dynamics of a super-Yang–Mills theory in a *hidden sector.* Whereas the perturbative scalar potential of the effective supergravity has flat directions—corresponding to an infinite vacuum degeneracy parameterized by massless scalars (*moduli*)—the effective theory of dynamical supersymmetry breaking lifts these flat directions and hence gives rise to *moduli stabilization.*

In section 2 we outline the field content that is present in the theories studied here. We describe the interpretations that are to be given to these fields in the context of string-inspired effective supergravity. We briefly review the *duality* that relates the chiral dilaton and linear dilaton formulations. We discuss the reasons why one might choose to work in the latter formulation in addition to the former. We comment on some instances in which the two formulations have confronted each other and seem to be at odds. A brief summary of previous works on the subject of supergravity with a linear dilaton is given. We relate these to our work and describe how our results supplement those that already exist. Finally, we define the superfield Lagrangian for which we have computed the component expansion.

[1] More specifically, BGW were concerned with the $E_8 \times E_8$ heterotic string. However, their effective supergravity description would work just as well for $spin(32)/Z_2$ constructions. The only issue there is to *hide* the *hidden sector.* Of course this is already a problem in the $E_8 \times E_8$ case due to the presence of *twisted sectors,* which couple subgroups of the two E_8's; for example, see the discussion in [8, 9].

Section 3 discusses some aspects of our results. We emphasize features of the component Lagrangian that we find interesting. We guide the reader to our main results—*lengthy formulae*—which are contained in appendices. Our conclusions are stated in section 4.

In appendix A we summarize our notations, conventions and abbreviations. In appendix B we outline the method of *projection* to component fields that is used in the $U(1)_K$ superspace formalism (see below). In appendix C we discuss the geometric identities of $U(1)_K$ superspace that are particularly useful in the computation of the component Lagrangian. In appendices D–F we present the lengthy formulae that comprise our principle results.

2. The effective theory

In this section, we introduce the reader to the class of theories that we intend to study. We relate our approach to the more familiar formulation with a chiral dilaton. We also discuss the motivations for working with a linear dilaton. Indeed, we believe that there exist instances where one might prefer to use a linear dilaton; these reasons are of a purely *practical* nature, as the two formalisms are equivalent provided they are *properly* related. (Specifically, the equations of motions should form equivalent systems and constraint equations in one formalism must find an equivalent expression in the other.)

2.1. Content and framework

Our intent in this subsection is only to present enough of a summary that the reader not familiar with these topics can understand the motivations and key concepts involved in the present work. For further details, the reader is invited to refer to the papers that we cite below; in particular the reviews [10, 11] are valuable for further study of formal issues of linear multiplets in the $U(1)_K$ formalism and papers [12, 13] provide nice reviews of the related superstring phenomenology.

The theory we consider is a variety of four-dimensional $N = 1$ supergravity. It contains the graviton, Yang–Mills gauge fields, matter fields and moduli fields, including a dilaton. Each of the matter and moduli fields (excepting the dilaton) may or may not be charged with respect to the Yang–Mills gauge group. All of the fields in the theory are introduced along with superpartners through $N = 1$ supermultiplets, by starting with the *Kähler $U(1)$ superspace* formalism (denoted by $U(1)_K$) [14–16]. The $U(1)_K$ approach has been reviewed in [11], *hereafter referred to as BGG*. The minimal supergravity multiplet $\left(e_m{}^a, \psi_m{}^\alpha, \bar{\psi}_{m\dot{\alpha}}, M, \overline{M}, b_a\right)$ is introduced through the superdeterminant E of the supervielbein $E_M{}^A$ and geometric relations in $U(1)_K$ superspace. Gauge multiplets $\left(a_{(a)m}, \lambda_{(a)\alpha}, \bar{\lambda}^{\dot{\alpha}}_{(a)}, \mathbf{D}_{(a)}\right)$ are introduced through vector superfields fixed to Wess–Zumino gauge, where (a) labels a basis of orthogonal generators of the gauge group. Except in a counterterm associated with an anomalous $U(1)$ factor, only corresponding chiral field strengths $\mathcal{W}^\alpha_{(a)}$ will appear explicitly in the superfield Lagrangian, due to the appearance of the Yang–Mills connection in the covariant derivatives of $U(1)_K$ superspace. Matter multiplets and all of the moduli multiplets except the dilaton are introduced through chiral superfields Φ^k, which have field content $\left(\phi^k, \chi^k_\alpha, F^k\right)$. The dilaton is introduced through a (modified) linear superfield L; its field content will be discussed below. We account for the leading effects of a strongly coupled *hidden sector* with condensing gauge group G_C through static (auxiliary) chiral superfields $U_{(a)}$ and Π^α. Here, $U_{(a)}$ is in correspondence with the operator $(\mathcal{W}\mathcal{W})_{(a)}$ in the unconfined theory; i.e., its lowest component (the $\theta = \bar{\theta} = 0$ part, denoted by $|_{(0,0)}) u_{(a)} = U_{(a)}|_{(0,0)}$ corresponds to the gaugino bilinear operator $(\lambda\lambda)_{(a)}$ which acquires a nonvanishing vev (*vev*), triggering *gaugino condensation* [18]. The lowest component of Π^α corresponds to a scalar operator of hidden sector matter fields which may also take a nonvanishing vev and play a role in the effective theory of supersymmetry breaking through the dynamics of the hidden sector.

2.2. Linear versus chiral dilaton

In the context of string-inspired supergravity, the four-dimensional dilaton is a composite of ten-dimensional fields dimensionally reduced to an effective four-dimensional theory. In Witten's classic reduction [17], one has a real scalar σ, which arises from the ten-dimensional graviton, the ten-dimensional dilaton ϕ and a 2-form field strength $\hat{f}_{mnp}$. The four-dimensional dilaton is given by

$$\frac{1}{g_s^2(\sigma, \phi)} = \mathrm{e}^{3\sigma}\phi^{-3/4}. \tag{2.1}$$

It is the vev of this quantity that determines the strength of gauge couplings. The effective supergravity Lagrangian naturally pairs this four-dimensional dilaton with a pseudoscalar D that is the universal axion; however, a duality transformation must be made to trade $\hat{f}_{mnp}$ for D:

$$\phi^{-3/2}\,\mathrm{e}^{6\sigma}\hat{f}_{mnp} \equiv \epsilon_{mnpq}\partial^q D. \tag{2.2}$$

Then the natural pairing is

$$s = \mathrm{e}^{3\sigma}\phi^{-3/4} + 3\mathrm{i}\sqrt{2}D \tag{2.3}$$

since for a Kähler potential with leading-order s dependence $K \ni -\ln(s + \bar{s})$ the standard chiral supergravity formulation yields the correct terms in the effective Lagrangian. When we promote $s \to S$, a chiral superfield, we have the chiral dilaton formulation. Due to the $N = 1$ supersymmetry, the complex field s has a superpartner which is the *dilatino*. The chiral multiplet formulation is used for reasons of familiarity and simplicity.

Instead of making the duality transformation (2.2), we can work with an $N = 1$ multiplet that already contains a 2-form field strength—the linear multiplet [3, 4, 19, 20]. Whereas in the leading-order effective Lagrangian it is straightforward to impose (2.2) and replace $\hat{f}_{mnp}$ by D, in a more general setting one finds that the corresponding duality transformations are difficult to perform explicitly [21–23]. If the beyond-leading-order Lagrangian is obtained from string theory, so that it already contains the 2-form field strength $\hat{f}_{mnp}$, it may be more practical to work with the linear multiplet and thus avoid intricacies that may be associated with the duality transformation.

A second issue arises when an effective theory of gaugino condensation is included as a mechanism for dynamical supersymmetry breaking. The chiral field strength superfields of the Yang–Mills group satisfy

$$(\mathcal{D}^2 - 24\overline{R})\,\mathrm{Tr}(\mathcal{W}\mathcal{W}) - (\overline{\mathcal{D}}^2 - 24R)\,\mathrm{Tr}(\overline{\mathcal{W}\mathcal{W}}) = \mathrm{t.d.} \tag{2.4}$$

where 't.d.' stands for a total derivative. To treat the condensate superfield $U \sim \mathrm{Tr}(\mathcal{W}\mathcal{W})$ as an ordinary chiral superfield (of $U(1)_K$ weight 2) fails to implement this constraint [24–26]. In the linear dilaton formulation, the chiral field strength emerges from the *modified linearity conditions:*

$$(\overline{\mathcal{D}}^2 - 8R)L = -\,\mathrm{Tr}(\mathcal{W}\mathcal{W}) \qquad (\mathcal{D}^2 - 8\overline{R})L = -\,\mathrm{Tr}(\overline{\mathcal{W}\mathcal{W}}). \tag{2.5}$$

When $U = \sum_{(a)\in G_C} U_{(a)}$ (where G_C is the condensing part of the gauge group) is introduced in (2.5) through

$$\mathrm{Tr}(\mathcal{W}\mathcal{W}) \to \mathrm{Tr}(\mathcal{W}\mathcal{W}) + U \tag{2.6}$$

then (2.4) is *automatically satisfied* for the $U_{(a)}$ [6]. Although this constraint *can* be imposed in the chiral dilaton multiplet formulation, it is more difficult. In this regard the linear multiplet has a practical advantage.

Of course one may ask (i) what evidence exists that would suggest (2.4) *should* be satisfied when $\mathrm{Tr}(\mathcal{W}\mathcal{W})$ is replaced by the interpolating field U, (ii) whether imposing this constraint has any important effects on the effective theory of dynamical supersymmetry breaking. These are certainly fair questions and we know of no clear answer to (i), except that it seems as the most reasonable assumption. We do have something to say about (ii).

In the present formalism, the condensate superfields $U_{(a)}$ are introduced as static chiral superfields. Their highest components $F_{U_{(a)}}$ (defined in (A.5)) thus appear only linearly in the component Lagrangian. However, a subtlety arises in deriving the corresponding equations of motion: an important constraint exists on the $F_{U_{(a)}}$ if we extend (2.4) to the condensates. That is, suppose we impose

$$(\mathcal{D}^2 - 24\overline{R})(\mathrm{Tr}(\mathcal{W}\mathcal{W}) + U) - (\overline{\mathcal{D}}^2 - 24R)(\mathrm{Tr}(\overline{\mathcal{W}\mathcal{W}}) + \overline{U}) = \text{t.d.} \tag{2.7}$$

Then it was noted in [5] that we have for the highest components $F_{U_{(a)}}$ the constraints

$$-\tfrac{1}{4}(\mathcal{D}^2 U_{(a)} - \overline{\mathcal{D}}^2 \overline{U}_{(a)}) = F_{U_{(a)}} - \overline{F}_{\overline{U}_{(a)}} = 4\mathrm{i}\nabla_m B^m_{(a)} + u_{(a)}\overline{M} - \overline{u}_{(a)}M. \tag{2.8}$$

Here $\nabla_m B^m_{(a)}$ is identified with the total derivative term in (2.7), while $u_{(a)}\overline{M} - \overline{u}_{(a)}M$ arises from the $24\overline{R}U - 24R\overline{U}$ part. When one varies the action with respect to the auxiliary fields $F_{U_{(a)}}$, it is crucial to respect (2.8) by first rewriting $F_{U_{(a)}}$ as

$$F_{U_{(a)}} = \tfrac{1}{2}\big(F_{U_{(a)}} + \overline{F}_{\overline{U}_{(a)}}\big) + 2\mathrm{i}\nabla_m B^m_{(a)} + \tfrac{1}{2}(u_{(a)}\overline{M} - \overline{u}_{(a)}M) \tag{2.9}$$

and the conjugate of this for $\overline{F}_{\overline{U}_{(a)}}$. (For example, this has been done in equation (2.21) of [6].) One then varies with respect to the *unconstrained* combination $F_{U_{(a)}} + \overline{F}_{\overline{U}_{(a)}}$.

The crucial thing to note is the last term on the right-hand side of (2.9). Generically, it has a nonvanishing vev when the scalar potential is minimized[2]. It is a supersymmetry breaking vev which couples to operators that appear with a coefficient $F_{U_{(a)}}$ in the Lagrangian. In particular, it can contribute to soft terms in the low energy effective Lagrangian. Note also that $u_{(a)}\overline{M} - \overline{u}_{(a)}M$ is anti-Hermitian whereas $F_{U_{(a)}} + \overline{F}_{\overline{U}_{(a)}}$ is Hermitian. Thus these operators couple to different parts of the operators that are coefficients of $F_{U_{(a)}}$ in the Lagrangian prior to the substitution (2.9).

On the other hand, if we had treated $U_{(a)}$ as an ordinary chiral superfield of $U(1)_K$ weight 2, we would have

$$\mathcal{D}^2 U_{(a)} - \overline{\mathcal{D}}^2 \overline{U}_{(a)} = \text{t.d.} \tag{2.10}$$

and therefore

$$F_{U_{(a)}} - \overline{F}_{\overline{U}_{(a)}} = \text{t.d.} \tag{2.11}$$

The supersymmetry breaking vev disappears on the right-hand side in this approach, leading to a different phenomenology. Incidentally, this also occurs in the approach taken in [22], where it was assumed that the highest component of the Chern–Simons superfield $\Omega_{(a)}$ (see below) vanishes. But this is nothing other than $F_{U_{(a)}}$, which as we have seen does not have a vanishing vev in the BGW approach if supersymmetry is broken. In fact, if we look at the perturbative

[2] Clearly, the phases of the condensates $u_{(a)}$ and the auxiliary scalar M are intimately involved in whether or not this term vanishes in the vacuum. Thus a more detailed study of the axionic background is necessary to understand its relevance. For example, see [6].

form of $\Omega_{(a)}$ as given in [22], there is no apparent reason why its highest component would not get a vev, since it is a scalar operator that solely contains strongly interacting fields.

One might think that it would be useful to relate the effective Lagrangian to that of the chiral dilaton formulation and compare in the global limit where the effective theory is known [27]. However, the last term on the right-hand side of (2.9) is implicitly suppressed by powers of $1/m_P$, the inverse reduced Planck mass, and is obviously a supergravity effect. (After all, it involves the auxiliary scalar M of supergravity.) Hence it would not appear in the theories of global supersymmetry. Nevertheless it is important in the present context because the soft term phenomenology will generally be affected by its presence.

In conventional supergravity coupled to super-Yang–Mills and a chiral dilaton, the chiral field strength superfields $\mathcal{W}^{\alpha}_{(a)}$ are introduced through an *F-density,* which necessitates a holomorphic metric $f_{(a)(b)}(S,\Phi)$; i.e., one has

$$\mathcal{L}_{\mathrm{YM}} = \int \mathrm{d}^4\theta \frac{E}{8R} f_{(a)(b)}(S,\Phi)(\mathcal{W}^{(a)}\mathcal{W}^{(b)}) + \mathrm{h.c.} \tag{2.12}$$

However, it is well known that this is not the unique local superfield Lagrangian through which the Yang–Mills field strength can be introduced; the possibility of a nonholomorphic metric is allowed if instead we introduce the chiral field strength superfields using a *D-density* Lagrangian [1]. Generically, the linear multiplet approach leads to a super-Yang–Mills Lagrangian that is equivalent to a combination of the holomorphic F-density and nonholomorphic D-density. On the other hand, it has been shown that the one-loop effective Lagrangian derived from heterotic orbifold models [28] is such that one can always write the super-Yang–Mills Lagrangian as a pure F-density [21]. In the linear dilaton formulation matched to the string theoretic calculations, one of course obtains a super-Yang–Mills Lagrangian that is just an F-density in the chiral dilaton formulation[3]. But, it would be interesting to formulate the corresponding conditions explicitly in the general context. The component expansion provided here may be of some aid in such an enterprise.

The advantages of the linear multiplet listed above suggest that in detailed model-building—which intends to go beyond leading order, implement gaugino condensation, and nonperturbative corrections to the dilaton Kähler potential—the linear dilaton is a practical tool. At the very least, it is worthwhile to have parallel studies in a different formulation which is supposed to be equivalent to the chiral dilaton. These are among the reasons for which BGW chose to work in this setting. It then becomes useful to have component field expansions that are general enough to handle the cases envisioned in semi-realistic applications. This is the motivation for the computation reported here.

To properly relate the linear dilaton to the chiral dilaton, the duality transformation should respect supersymmetry. One approach is to perform the duality transformation at the superfield level [4]. For global supersymmetry, this duality has been reviewed in section 2 of [22] and [10]. For the locally supersymmetric (supergravity) case in the $U(1)_K$ formalism, a review of chiral-linear duality has been given (briefly) in BGG section 5.5. The duality in the superconformal approach has been discussed in [21, 22].

For illustrative purposes, consider the Lagrangian

$$\mathcal{L} = \int \mathrm{d}^4\theta\, E \left[-2 + f(L) + \frac{1}{3}(L+\Omega)(S+\bar{S}) \right]. \tag{2.13}$$

Here L is treated as a real superfield which is otherwise unconstrained; that is, $\mathcal{L}$ is supposed to represent a *first-order formulation* of the *target theory.* S is a chiral superfield. Ω is real

[3] The component expansions given below are general enough to accommodate either situation; to obtain only an F-density requires that arbitrary functionals that appear here satisfy certain conditions.

and is the *Chern–Simons superfield* [29] (see also [21] or appendix F.3 of BGG). It satisfies the constraints

$$(\overline{\mathcal{D}}^2 - 8R)\Omega = \mathrm{Tr}(\mathcal{W}\mathcal{W}) \qquad (\mathcal{D}^2 - 8\overline{R})\Omega = \mathrm{Tr}(\overline{\mathcal{W}\mathcal{W}}). \tag{2.14}$$

The superfield equations of motion obtained from (2.13) yield the duality

$$\frac{L}{1 + f(L)} = \frac{1}{S + \bar{S}} \tag{2.15}$$

together with the modified linearity conditions (2.5). We denote the $\theta = \bar{\theta} = 0$ (*lowest*) components as

$$L|_{(0,0)} = \ell \qquad S|_{(0,0)} = s \qquad \bar{S}|_{(0,0)} = \bar{s}. \tag{2.16}$$

Above we mentioned that in the chiral formulation the string coupling g_s is determined by the real part of s; the duality relation (2.15) gives a corresponding meaning to ℓ:

$$g_s^2 = \frac{2}{s + \bar{s}} = \frac{2\ell}{1 + f(\ell)}. \tag{2.17}$$

Further details on the duality relations—for the case of local supersymmetry in the $U(1)_K$ formalism, including the other component fields—may be found in [15, 20].

A comparison of phenomenological implications of either formulation is given in [6]. In this work it was found that the Kähler moduli of the underlying theory are not stabilized at their *self-dual points* in the chiral dilaton approach, whereas the stabilization does occur at the self-dual points in the linear dilaton approach. It was argued that the disparate results originate from an explicit S dependence in the effective superpotential of the chiral dilaton formulation.

However, if one starts with a linear dilaton and performs a duality transformation analogous to (2.13), it *must* be that one obtains a chiral dilaton formulation which is equivalent *on-shell*. That is, the first-order formalism *ensures* that the equations of motion for the linear dilaton supergravity are equivalent to the equations of motion for the chiral dilaton supergravity, and that the equivalence is established through the superfield redefinition that is obtained through the duality transformation—the generalization of (2.15). The stabilization of moduli is studied through minimization of the scalar potential. But this is nothing but a study of solutions to the equations of motion in the infrared limit, neglecting all fields with nonzero spin. Since the equations are equivalent in the two approaches, they must yield equivalent solutions. Thus it is our opinion that some subtlety must have been overlooked in performing the duality transformation for the theory studied in [6]. In that case the target theory was more complicated than the illustrative example (2.13). We intend to return to this issue in a future publication. A full component Lagrangian may shed some light on this issue, since it allows us to study the duality transformations at the component field level.

2.3. Antecedants

The effective supergravity discussed here is an extension of the BGW effective theory [5–7], which does not include an anomalous $U(1)$ factor (hereafter denoted by $U(1)_X$) in the gauge group. A $U(1)_X$ is a generic feature of semi-realistic string constructions; for example, in [9] it was found that 168 of 175 models had a $U(1)_X$. The associated anomaly is cancelled by a *Green–Schwarz (GS) counterterm* [30, 31], as will be discussed below. The linear multiplet formulation provides an elegant description of the effective supergravity that results, as has been discussed in [32]. Indeed [32–34] aim to address the modifications to the BGW effective theory in the presence of an anomalous $U(1)$. However, in none of these references is the full fermionic Lagrangian presented; only the gravitino and gaugino effective masses have been computed [6, 7]. Moreover, we allow for unconfined matter to couple to the (auxiliary) hidden matter condensate superfields Π^α. This is important for the stabilization of flat directions in the presence of an anomalous $U(1)$ factor, so-called *D-moduli* [35].

Fermion terms of component Lagrangians in the linear dilaton formulation have previously been computed by authors other than BGW to varying degrees.

In [16], Adamietz *et al* obtained *all* the fermionic terms. However, no superpotential was included in the Lagrangian, a GS counterterm for a $U(1)_X$ was not included, and an effective theory of gaugino condensation was not explicitly added. Adamietz *et al* also made the simplifying assumption that the Kähler potential for the linear multiplet is $k(L) = \alpha \ln L$, which is equivalent to the assumption $K(S, \overline{S}) \propto -\ln(S + \overline{S})$ in the chiral dilaton approach. Stabilization of the dilaton sometimes requires a more general function, such as will be studied here.

In [22], Derendinger *et al* only gave some of the fermion terms; in particular gaugino bilinears. Their treatment of gaugino condensation differs from that of BGW in some important ways, as will be discussed below; these differences affect predictions for soft supersymmetry breaking operators in the low energy effective theory. Also, a GS counterterm for a $U(1)_X$ was not included in the effective theory. Derendinger *et al* use the superconformal tensor calculus [36] to obtain the component Lagrangian, whereas we use $U(1)_K$ superspace. We believe that it is useful to have results in both formalisms.

Various other limiting assumptions were made in these previous works which have not been made here. Thus our calculation can accommodate a more general set of circumstances and exhibits possible couplings that were not accounted for in previous works.

2.4. The Lagrangian

In this paper a very general Kähler potential is assumed; it is the same as for BGW [5–7]:

$$K = k(L) + G(\Phi, \overline{\Phi}) \qquad k(L) = \ln L + g(L). \tag{2.18}$$

Here $g(L)$ is left arbitrary in our calculations, though we have in mind the sort of nonperturbative corrections that are expected based on general arguments [37] and string duality [38]. Indeed these sorts of corrections have been used by BGW and others to stabilize the dilaton at weak coupling (i.e., $g_s^2 \lesssim 1$ in (2.17) in a scheme that has come to be known as *Kähler stabilization* [5–7, 39–42].

The Lagrangian consists of several pieces[4]:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{VY}} + \mathcal{L}_{\text{thr}} + \mathcal{L}_{\text{GS}}^{0} + \mathcal{L}_{\text{GS}}^{X}. \tag{2.19}$$

The first piece contains the usual kinetic terms for all the fields, and is written in the $U(1)_K$ superspace formalism as follows:

$$\mathcal{L}_{\text{kin}} = \int \mathrm{d}^4\theta\, E[-2 + f(L)]. \tag{2.20}$$

The function $f(L)$ is chosen such that a canonical Einstein term $-\frac{1}{2}\mathcal{R}$ (where $\mathcal{R}$ is the Ricci scalar) is obtained in the component expansion. With reference to the Kähler potential (2.18), the condition for this to be true is that

$$Lg'(L) = f(L) - Lf'(L) \tag{2.21}$$

where $g'(L) = \mathrm{d}g(L)/\mathrm{d}L$, etc. An elementary discussion of how this condition occurs can be found in section 5.4 of BGG, where their function F is related to the f used here according to $F = (2 - f)/3$.

[4] See [6] for further discussion on the significance of each term.

The usual superpotential term is included:

$$\mathcal{L}_{\text{pot}} = \int \mathrm{d}^4\theta \frac{E}{2R} \mathrm{e}^{K/2} W(\Phi, \Pi) + \text{h.c.} \tag{2.22}$$

We remind the reader that the chiral superfields Π^α are static fields corresponding to matter condensates of the hidden sector. Thus they do not appear in (2.18), but it is important to include them in (2.22).

In addition to this *bare* superpotential, we have the Veneziano–Yankielowicz effective superpotential [43], with suitable modifications suggested by Taylor [44]:

$$\mathcal{L}_{\text{VY}} = \int \mathrm{d}^4\theta \frac{E}{8R} \sum_{(a)\in G_C} U_{(a)} \left[b'_{(a)} \ln(\mathrm{e}^{-K/2} U_{(a)}/\mu^3) + \sum_\alpha b^\alpha_{(a)} \ln \left(A^\alpha_{(a)}(\Phi)\Pi^\alpha \right) \right] + \text{h.c.} \tag{2.23}$$

The gaugino condensate superfields $U_{(a)}$ appear explicitly here. When they are integrated we obtain the usual nonperturbative superpotential [27] induced by instanton effects[5], only coupled to supergravity in the present context; to match to the globally supersymmetric results one should take the decoupling limit $m_P \to \infty$. The coefficients $b'_{(a)}$ and $b^\alpha_{(a)}$ are constrained by a matching to the weak coupling quantum anomalies. A further discussion can be found, for example, in [6].

Massive string states can yield threshold corrections to the effective theory below the string scale. The well-known corrections associated with $N = 2$ sectors in orbifold compactifications of the heterotic string are given by [28, 45]

$$\mathcal{L}_{\text{thr}} = \sum_I \int \mathrm{d}^4\theta \frac{E}{8R} \left[\sum_{(a)\notin G_C} b^I_{(a)} (\mathcal{W}\mathcal{W})_{(a)} + \sum_{(a)\in G_C} b^I_{(a)} U_{(a)} \right] \ln \eta^{-2}(T^I) + \text{h.c.} \tag{2.24}$$

where the coefficients $b^I_{(a)}$ are determined by explicit string calculations.

Quantum anomalies that arise from the terms so far described are cancelled by Green–Schwarz (GS) counterterms. The first involves a real function S—not to be confused with the chiral dilaton of the discussion above—which we will refer to as the *GS potential*. We restrict S to be a function of chiral superfields: $S = S(\Phi, \overline{\Phi})$. Its purpose is to cancel target-space duality anomalies. Its superfield expression is

$$\mathcal{L}^0_{\text{GS}} = \int \mathrm{d}^4\theta\, E L S. \tag{2.25}$$

The precise form of S can only be obtained from a detailed understanding of the full anomaly structure of the effective supergravity and how it is cancelled in the underlying string theory. The *modular anomaly* associated with $SL(2, Z)^3$ transformations on *Kähler moduli* associated with the complex planes in orbifold compactifications of the heterotic string is well known. It is partially cancelled by, for example, a choice of $S \propto G$, where G is identified in (2.18). However, a richer anomaly structure is anticipated on the basis of one-loop supergravity calculations [46, 47], and so we leave S arbitrary in our component expansion. We note that since the GS counterterm potential S is left in a rather general form, our component field expansions do not assume modular invariance; that is, we can accommodate models where violations of modular invariance are envisioned, due to nonperturbative effects in the underlying string theory. On the other hand, exact modular invariance can also be imposed with an appropriate choice for S.

5 Indeed, one can argue that $U_{(a)}$ have masses of order the condensation scale and *should* be integrated to obtain the effective theory below that scale. We thank Erich Poppitz for a remark in this regard.

The second GS counterterm is associated with the anomalous $U(1)_X$, with a corresponding vector superfield V_X. It is given by

$$\mathcal{L}_{\mathrm{GS}}^{X} = \int \mathrm{d}^4\theta\, E L \delta_X V_X. \tag{2.26}$$

This addition to the BGW effective theory has been the subject of recent work [32–34]. There it was shown how to fix to unitary gauge and integrate the modes that acquire large masses when the Fayet–Iliopoulos (FI) term that arises from (2.26) spontaneously breaks $U(1)_X$ at a high scale.

We have omitted the perturbative one-loop effective quantum correction to the Lagrangian. Related expressions have been studied by various groups: by Derendinger *et al* using superconformal methods [21], by Bagger *et al* using a component field approach [48], by Gaillard *et al* in $U(1)_K$ superspace [49]. However, all of these calculations involve various simplifying assumptions on the form of the bare Lagrangian compared to what is given here. Furthermore, in the calculation of [49] there exist some uncertainties in the precise form of the chiral projection operator P_χ employed there. In principle, the one-loop effective quantum correction to the Lagrangian can be derived from $\mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{pot}}$ by a one-loop computation using, say, Pauli–Villars regularization [46]. In fact, much of this calculation has been performed in [46] for the class of Lagrangians studied here. One possible motivation for the present work is to fill in the fermionic details of the component Lagrangian needed to complete the one-loop computation.

3. Aspects of the component Lagrangian

Our results for $\mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{VY}} + \mathcal{L}_{\mathrm{thr}} + \mathcal{L}_{\mathrm{GS}}^{X}$ are given in appendix D. For this part of $\mathcal{L}$ the component field expansion is straightforward, except for a certain subtlety that arises in $\mathcal{L}_{\mathrm{GS}}^{X}$. This has to do with the evaluation of spinorial derivatives acting on the $U(1)_X$ vector superfield V_X. Here it is important to properly account for the conventions of BGG for the solution of superspace Bianchi identities; details are given in appendix A.

The superpotential Lagrangian (D.1) contains the usual terms that are present in chiral supergravity; of course a mixing with the dilaton occurs due to the ℓ dependence in the $\mathrm{e}^{K/2}$ pre-factor for these terms[6]. In addition we have pieces explicitly associated with the linear supermultiplet:

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{\mathrm{pot}}^{L} = \mathrm{e}^{K/2}\Bigg\{ &-\frac{1}{4}W(k'' + k'^2)(\varphi\varphi) - \frac{1}{\sqrt{2}}(W_k + WG_k)\,k'(\varphi\chi^k) \\ &+ Wk'\left[\frac{1}{4}\bar{u} - \frac{1}{4}\mathrm{Tr}(\bar{\lambda}\bar{\lambda}) + \frac{1}{3}\overline{M}\ell + \frac{\mathrm{i}}{2}(\bar{\psi}^m\bar{\sigma}_m\varphi)\right]\Bigg\} + \mathrm{h.c.}\end{aligned} \tag{3.1}$$

The bosonic terms were previously studied in the works of BGW. Note that the (effective) dilatino and gaugino masses receive contributions from the bilinears $\varphi\varphi$ and $\bar{\lambda}\bar{\lambda}$ that appear in (3.1). The bilinear $\varphi\chi^k$ which mixes the dilatino φ with matter fermions χ^k is a feature that deserves further study[7]. In particular, the presence of large vevs generally leads to important effects that would arise from the $\varphi\chi^k$ bilinear. For example, it is common in semi-realistic string models for exotic states to be removed at a high scale through large effective masses generated by FI-induced vevs,

$$m_{ij} \sim \tfrac{1}{2}\langle \mathrm{e}^{K/2} W_{ij}\rangle \lesssim \mathcal{O}(0.1)m_P \tag{3.2}$$

[6] We remind the reader of the dilaton (ℓ) dependent contribution $k(\ell)$ to the Kähler potential (2.18); also note that $k' = \partial k/\partial \ell$, etc below.

[7] The coupling of the dilatino to the gravitino can be eliminated with the 'gauge' choice $(\bar{\psi}^m\bar{\sigma}_m)^\alpha = 0$.

where m_P is the reduced Planck mass. This implies effective couplings in (3.1) of the form

$$-\sqrt{2}\frac{m_{ij}}{m_P}k'(\ell)\phi^i(\varphi\chi^j). \tag{3.3}$$

The implications of such couplings for the cosmology associated with the dilaton, dilatino and heavy matter states present an interesting topic for further study.

A feature that is special to the fermionic terms appears from the effective theory of gaugino condensation. This is related to the auxiliary fermions $\Lambda^\alpha_{(a)}$ contained in the gaugino condensate superfields $U_{(a)}$; see (A.5). When these fields are eliminated by their equations of motion, we obtain their contribution to the Lagrangian:

$$\begin{aligned}\frac{1}{e}\mathcal{L}(\Lambda) &= \sum_{(a)\in G_C}\frac{b'_{(a)}}{8u_{(a)}}(\Lambda\Lambda)_{(a)} + \text{h.c.}\\ &= \sum_{(a)\in G_C}\frac{2u_{(a)}}{b'_{(a)}}\big\{(f^{(a)})^2(\varphi\varphi) + \hat{f}^{(a)}_k\hat{f}^{(a)}_\ell(\chi^k\chi^\ell)\\ &\quad + (\tilde{f}^{(a)})^2(\bar{\psi}_m\bar{\sigma}^m\sigma^n\bar{\psi}_n) + 2f^{(a)}\hat{f}^{(a)}_k(\varphi\chi^k)\\ &\quad + 2\mathrm{i}\tilde{f}^{(a)}\big[f^{(a)}(\bar{\psi}_m\bar{\sigma}^m\varphi) + \hat{f}^{(a)}_k(\bar{\psi}_m\bar{\sigma}^m\chi^k)\big]\big\} + \text{h.c.}\end{aligned} \tag{3.4}$$

$$f^{(a)} = \frac{1}{4\sqrt{2}}\left[b'_{(a)}k' - f'' - \left(\frac{k'+\ell k''}{1-\frac{1}{3}\ell k'}\right)k' - 2k'' - \left(\frac{k'+3k''\ell}{k'\ell-3}\right)k'\right] \tag{3.5}$$

$$\hat{f}^{(a)}_k = \frac{1}{4}\big(b'_{(a)}G_k - h^{(a)}_k - S_k\big) \tag{3.6}$$

$$\tilde{f}^{(a)} = \frac{1}{8\sqrt{2}}\big[2b'_{(a)}\ln\big(\mathrm{e}^{1-\frac{K}{2}}u_{(a)}/\mu^3\big) + 2h^{(a)} + f' + k' + S\big]. \tag{3.7}$$

The (holomorphic) function $h^{(a)}(\phi,\pi)$ is defined in (B.6), and

$$h^{(a)}_k\chi^k = \frac{\partial h^{(a)}}{\partial\phi^k}\chi^k + \frac{\partial h^{(a)}}{\partial\pi^\alpha}\chi^\alpha. \tag{3.8}$$

Here $\sqrt{2}\chi^{\alpha\beta} = \mathcal{D}^\beta\Pi^\alpha|_{(0,0)}$ is a further auxiliary spinor, contained in the matter condensate superfield, which can likewise be eliminated using its equations of motion.

As can be seen from (3.4), we obtain a *soft mass* for the dilatino; it is roughly the order of the supersymmetry breaking scale $u_{(a)}/m_P^2 \sim 1$ TeV. Furthermore, matter fermions get a soft mass contribution, which includes mixing with the dilatino, suppressed by $\langle\hat{f}^{(a)}_k\rangle/m_P$. These must be singlets under the *standard model* gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ in order for this effect to matter. To see this note that $\hat{f}^{(a)}_k$ transforms as the conjugate of χ^k. This implies that $\langle\hat{f}^{(a)}_k\rangle/m_P \lesssim 10^{-15}$ if χ^k is charged under the standard model. Thus these terms are relevant only in extended models, such as the *non-minimal supersymmetric standard model (NMSSM),* or models with an *inflatino,* etc. Going to the gauge $\bar{\psi}_m\bar{\sigma}^m \equiv 0$ demonstrates that the above terms do not contribute to the gravitino mass.

In the language of [22], the auxiliary spinors $\Lambda_{(a)}$ and $\overline{\Lambda}_{(a)}$ correspond to the next-to-highest components of the Chern–Simons superfield $\Omega_{(a)}$. However, the authors of [22] set these components identically to zero (cf their equation (3.29)); hence, the above contributions to soft fermion masses do not appear in their results. This is one of the ways in which our results generalize previous work.

4. Conclusions

In this paper, we have carried out the calculation of the component Lagrangian for supergravity coupled to gauged matter and a linear dilaton multiplet. We have reviewed the reasons why this alternative to the chiral dilaton formulation might be useful. We have commented on previous work that exists in the literature and have explained the ways in which the results presented here generalize those that have appeared before. We have offered ways in which these results might be put to use.

In particular, we believe that further details of the duality between the linear dilaton and the chiral dilaton should be explored. Since some results in the literature are at odds it would appear that a more careful comparison at the component field level may uncover the errors which we suppose have led to these discrepancies. At the same time, holomorphy prevents corrections to the superpotential that involve the linear multiplet and these must arise as exact symmetries in the chiral formulation. It would be interesting to make the connection more precise. The component expansion provided here makes that possible since the duality transformations can be checked at the component field level.

Kähler stabilization involves deviations of the dilaton Kähler potential from the leading-order form. Such corrections are easily encoded with the modular invariant L, whereas they require the modified dilaton multiplet S' which mixes S and T^I in the chiral formulation[8]. This is no real impediment, but it does make the two frameworks difficult to relate; we can imagine that each *naturally* probes some different regions of *parameter space* for these nonperturbative corrections. A further study of the duality transformations is required to determine the extent to which a representative coverage is achieved in either scheme.

We have also pointed out some of the peculiarities of the component Lagrangian. The fact that L cannot appear in the superpotential leads to special constraints that yield a more restrictive phenomenology if they are respected. Whereas they can be imposed in the chiral dilaton formulation, they 'fall out' in the present work. This is a nice feature because in a certain sense it automates model-building. We have also pointed out how the effective theory of dynamical supersymmetry breaking is impacted by the fact that the gaugino condensate superfield is obtained from L through Bianchi identities; in particular, we showed how this can impact the soft term phenomenology of the low energy theory.

As can be seen, several details of the effective theory, and its relation to other formulations, remain to be explicitly sorted out. We do not expect any remarkable things to be found through further exploration of this duality, but we anticipate that complete agreement between the formulations will emerge. As this goal is achieved, the related phenomenological studies will become increasingly reliable and accurate. Furthermore, the situations that can be studied easily will be enlarged by the availability of component Lagrangians that are more general and can thus accommodate a greater variety of assumptions at the superfield level.

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[8] That is, one would typically introduce nonperturbative corrections to the dilaton Kähler potential in a duality invariant way; see for example [21] for a definition of S'.

Appendix A. Notation and conventions

The linear superfield L is defined to satisfy modified linearity conditions such that

$$(\overline{\mathcal{D}}^2 - 8R)L = -U - \mathrm{Tr}(\mathcal{W}\mathcal{W}) \qquad (\mathcal{D}^2 - 8\overline{R})L = -\overline{U} - \mathrm{Tr}(\overline{\mathcal{W}\mathcal{W}}) \tag{A.1}$$

$$[\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot\alpha}]L = 4LG_{\alpha\dot\alpha} + 2B_{\alpha\dot\alpha} + 2\,\mathrm{Tr}(\mathcal{W}_\alpha \overline{\mathcal{W}}_{\dot\alpha}) \tag{A.2}$$

where we abbreviate sums over G_C (condensing parts of the gauge group) and non-G_C parts of the gauge group by

$$\mathrm{Tr}(\mathcal{W}\mathcal{W}) \equiv \sum_{(a)\notin G_C} (\mathcal{W}\mathcal{W})_{(a)} \qquad U \equiv \sum_{(a)\in G_C} U_{(a)} \quad \text{etc.} \tag{A.3}$$

In what follows, we adopt the conventions and notation of BGG for the definitions of component fields in terms of $\theta = \bar\theta = 0$ parts (denoted by $|_{(0,0)}$) of spinorial derivatives $(\mathcal{D}_\alpha, \overline{\mathcal{D}}^{\dot\alpha}$, etc) of superfields, with the exception that we denote the dilatino according to

$$\varphi_\alpha \equiv \mathcal{D}_\alpha L|_{(0,0)} \qquad \bar\varphi^{\dot\alpha} \equiv \overline{\mathcal{D}}^{\dot\alpha} L|_{(0,0)}. \tag{A.4}$$

In addition, we define the component fields

$$u_{(a)} = U_{(a)}|_{(0,0)} \qquad \Lambda_{(a)\alpha} = \frac{1}{\sqrt{2}}\mathcal{D}_\alpha U_{(a)}|_{(0,0)} \qquad F_{U_{(a)}} = -\frac{1}{4}\mathcal{D}^2 U_{(a)}|_{(0,0)} \tag{A.5}$$

and corresponding conjugates. We also have in the notation of (A.3)

$$u = \sum_{(a)\in G_C} u_{(a)} \qquad \Lambda_\alpha = \sum_{(a)\in G_C} \Lambda_{(a)\alpha} \qquad F_U = \sum_{(a)\in G_C} F_{U_{(a)}}. \tag{A.6}$$

For the (auxiliary) matter condensate superfields Π^α we have component fields

$$\pi^\alpha = \Pi^\alpha|_{(0,0)} \qquad \chi^\alpha_\beta = \frac{1}{\sqrt{2}}\mathcal{D}_\beta \Pi^\alpha|_{(0,0)} \qquad F^\beta = -\frac{1}{4}\mathcal{D}^2\Pi^\alpha|_{(0,0)} \tag{A.7}$$

where α should not be confused with a spinor index.

A semicolon denotes the usual Kähler covariant differentiation on the complex scalar manifold; e.g.,

$$W_{k;\ell} = W_{k\ell} - \Gamma^m_{k\ell} W_m \qquad \Gamma^m_{k\ell} = G^{m\bar m} G_{k\bar m \ell}. \tag{A.8}$$

In addition to the usual gauge and spacetime reparameterization covariance, Kähler covariance and $U(1)_K$ covariance are included in the covariant derivatives that appear in the component expansions; e.g., for the fermionic components of chiral superfields we have

$$\mathcal{D}_m \chi^k_\alpha = \partial_m \chi^k_\alpha - \omega_{m\alpha}{}^\beta \chi^k_\beta - A_m \chi^k_\alpha - \mathrm{i} a^{(a)}_m (T_{(a)}\chi_\alpha)^k + \chi^i \Gamma^k_{ij} \mathcal{D}_m \phi^j \tag{A.9}$$

and for the dilatino we have

$$\mathcal{D}_m \varphi^\alpha = \partial_m \varphi^\alpha + \varphi^\beta \omega_{m\beta}{}^\alpha - \varphi^\alpha A_m. \tag{A.10}$$

Here, $\omega_{m\alpha}{}^\beta$ is the usual spin connection, $a^{(a)}_m$ is the Yang–Mills connection, A_m is the $U(1)_K$ connection, and the Kähler connection Γ^k_{ij} is defined in (A.8). The coefficient of the A_m term in $\mathcal{D}_m$ depends on the $U(1)_K$ weight of the field on which it acts[9]. The component field expansion for $A_m \equiv A_m|_{(0,0)}$ is given by

$$A_m|_{(0,0)} = -\frac{\mathrm{i}}{2}b_m + \frac{1}{4}G_k \mathcal{D}_m \phi^k - \frac{1}{4}G_{\bar k}\mathcal{D}_m \bar\phi^{\bar k} + \frac{\mathrm{i}}{4}G_{k\bar k}(\chi^k \sigma_m \bar\chi^{\bar k}) - \frac{\mathrm{i}}{8}k''(\bar\varphi\bar\sigma_m\varphi) + \frac{\mathrm{i}}{6}k'\ell b_m$$
$$- \frac{\mathrm{i}}{4}k' B_m - \frac{\mathrm{i}}{8}k'\,\mathrm{Tr}(\bar\lambda\bar\sigma_m\lambda) + \frac{1}{8}k'(\psi_m\varphi) - \frac{1}{8}k'(\bar\psi_m\bar\varphi). \tag{A.11}$$

[9] See section 4 of BGG for a fuller specification and explanation of covariant derivatives in the present formalism.

We have checked that our expression for $A_m|_{(0,0)}$ is equivalent to (BGG-E.3.4) in the special case of $k(L) = \alpha \ln L$; in this calculation (BGG-5.2.20) and (BGG-5.3.7) are especially useful; furthermore, a typo in (BGG-E.3.4) must be corrected—the 'level' factor of k (not to be confused with the functional $k(L)$) should be absent on the ${}^*h_m$ term that appears in (BGG-E.3.4).

We evaluate the terms of $\mathcal{L}_{\mathrm{GS}}^X$ and its spinorial derivatives in Wess–Zumino (WZ) gauge:

$$V_X|_{(0,0)} \underset{\mathrm{WZ}}{=} \mathcal{D}_\alpha V_X|_{(0,0)} \underset{\mathrm{WZ}}{=} \overline{\mathcal{D}}_{\dot\alpha} V_X|_{(0,0)} \underset{\mathrm{WZ}}{=} \mathcal{D}_\alpha \mathcal{D}_\beta V_X|_{(0,0)} \underset{\mathrm{WZ}}{=} \overline{\mathcal{D}}_{\dot\alpha} \overline{\mathcal{D}}_{\dot\beta} V_X|_{(0,0)} \underset{\mathrm{WZ}}{=} 0. \tag{A.12}$$

To evaluate the component field expansions of the spinorial derivatives of V_X, we must be careful to use the conventions of BGG for the solution to the superspace Bianchi identities, and *not* those of, for example, Wess and Bagger [2]. Taking this into account, we find that

$$\begin{aligned}
\mathcal{D}_\alpha \overline{\mathcal{D}}_{\dot\alpha} V_X|_{(0,0)} &\underset{\mathrm{WZ}}{=} -a_{X\alpha\dot\alpha} \\
\mathcal{D}_\alpha \overline{\mathcal{D}}^2 V_X|_{(0,0)} &\underset{\mathrm{WZ}}{=} 4\mathrm{i}\lambda_{X\alpha} + 2\mathrm{i}a_{Xm}(\sigma^n\bar\sigma^m\psi_n)_\alpha \\
\mathcal{D}^2 \overline{\mathcal{D}}^2 V_X|_{(0,0)} &\underset{\mathrm{WZ}}{=} 8\mathbf{D}_X + \tfrac{16}{3}b^m a_{Xm} - 4a_{Xn}(\psi^m\sigma^n\bar\psi_m) - 8\mathrm{i}\mathcal{D}^m a_{Xm} \\
&\quad + 4(\bar\lambda_X\bar\sigma^m\psi_m) - 4(\bar\psi^m\bar\sigma_m\lambda_X).
\end{aligned} \tag{A.13}$$

These expressions are sufficient to compute $\mathcal{L}_{\mathrm{GS}}^X$, when combined with other identities given here and in BGG.

It proves convenient to introduce the following abbreviations:

$$\Delta_m\phi^k = e_m{}^a\mathcal{D}_a\Phi^k|_{(0,0)} = \mathcal{D}_m\phi^k - \frac{1}{\sqrt{2}}(\psi_m\chi^k) \tag{A.14}$$

$$\Delta_m\bar\phi^{\bar k} = e_m{}^a\mathcal{D}_a\overline{\Phi}^{\bar k}|_{(0,0)} = \mathcal{D}_m\bar\phi^{\bar k} - \frac{1}{\sqrt{2}}(\bar\psi_m\bar\chi^{\bar k}) \tag{A.15}$$

$$\Delta_m\ell = e_m{}^a\mathcal{D}_a L|_{(0,0)} = \partial_m\ell - \tfrac{1}{2}(\psi_m\varphi) - \tfrac{1}{2}(\bar\psi_m\bar\varphi) \tag{A.16}$$

$$\mathrm{i}\hat f_{(a)nm} = \mathrm{i}f_{(a)nm} + (\psi_n\sigma_m\bar\lambda_{(a)}) + (\bar\psi_n\bar\sigma_m\lambda_{(a)}). \tag{A.17}$$

Because of its length, we find it convenient to abbreviate $\mathcal{D}^2(\mathcal{W}\mathcal{W})_{(a)}|_{(0,0)}$ below. It is straightforward to obtain $\mathcal{D}^2(\mathcal{W}\mathcal{W})_{(a)}|_{(0,0)}$ from (BGG-4.5.25) if one makes the identification $f_{(r)(s)} \equiv -16$ for the functional $f_{(r)(s)}$ that appears there:

$$\begin{aligned}
\mathcal{D}^2(\mathcal{W}\mathcal{W})_{(a)}|_{(0,0)} = {} & 2f_{(a)}^{mn}f_{(a)mn} + \mathrm{i}\epsilon^{mnpq}f_{(a)mn}f_{(a)pq} + 8\mathrm{i}(\lambda\sigma^m\mathcal{D}_m\bar\lambda)_{(a)} \\
& - 4\mathbf{D}_{(a)}^2 + 4\overline{M}(\lambda\lambda)_{(a)} - 4(\lambda_{(a)}\sigma^m\bar\psi_m)\mathbf{D}_{(a)} \\
& - 4\mathrm{i}[(\psi_m\sigma^{pq}\sigma^m\bar\lambda_{(a)}) + (\bar\psi_m\bar\sigma^{pq}\bar\sigma^m\lambda_{(a)}) - (\bar\psi_m\bar\sigma^m\sigma^{pq}\lambda_{(a)})]f_{(a)pq} \\
& - 2[(\psi_m\sigma^{pq}\sigma^m\bar\lambda_{(a)}) + 2(\bar\psi_m\bar\sigma^{pq}\bar\sigma^m\lambda_{(a)}) - (\bar\psi_m\bar\sigma^m\sigma^{pq}\lambda_{(a)})] \\
& \times [(\psi_p\sigma_q\bar\lambda_{(a)}) + (\bar\psi_p\bar\sigma_q\lambda_{(a)})].
\end{aligned} \tag{A.18}$$

It is also useful to abbreviate the following components of the superspace torsion,

$$\begin{aligned}
T_{cb}{}^\alpha|_{(0,0)} = {} & \frac{1}{2}e_b{}^m e_c{}^n\left(\mathcal{D}_n\psi_m^\alpha - \mathcal{D}_m\psi_n^\alpha\right) + \frac{\mathrm{i}}{12}\left[e_c{}^m(\psi_m\sigma_n\bar\sigma_b)^\alpha - e_b{}^m(\psi_m\sigma_n\bar\sigma_c)^\alpha\right]b^n \\
& - \frac{\mathrm{i}}{12}\left[e_c{}^m(\bar\psi_m\bar\sigma_b)^\alpha - e_b{}^m(\bar\psi_m\bar\sigma_c)^\alpha\right]M
\end{aligned} \tag{A.19}$$

$$T_{cb\dot{\alpha}}|_{(0,0)} = \frac{1}{2}e_b{}^m e_c{}^n(\mathcal{D}_n\bar{\psi}_{m\dot{\alpha}} - \mathcal{D}_m\bar{\psi}_{n\dot{\alpha}}) - \frac{\mathrm{i}}{12}\left[e_c{}^m(\bar{\psi}_m\bar{\sigma}_n\sigma_b)_{\dot{\alpha}} - e_b{}^m(\bar{\psi}_m\bar{\sigma}_n\sigma_c)_{\dot{\alpha}}\right]b^n$$
$$- \frac{\mathrm{i}}{12}\left[e_c{}^m(\psi_m\sigma_b)_{\dot{\alpha}} - e_b{}^m(\psi_m\sigma_c)_{\dot{\alpha}}\right]\overline{M} \tag{A.20}$$

as can be found in (BGG-4.1.31) and (BGG-4.1.32).

Appendix B. Projection to component fields

If Ω is a real superfield of $U(1)_K$ weight zero, then we may use (BGG-D.1.10) to integrate by parts in superspace and obtain

$$\mathcal{L}_\Omega \equiv \int \mathrm{d}^4\theta\, E\Omega = \int \mathrm{d}^4\theta \frac{E}{2R}\hat{r}_\Omega + \mathrm{h.c.} \qquad \text{where} \quad \hat{r}_\Omega \equiv -\frac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\Omega. \tag{B.1}$$

Note that $\hat{r}_\Omega$ is the *chiral projection* of Ω. We use this technique to convert the integrals of (2.24) to the form (B.1). Doing so we have

$$\hat{r}_{\mathrm{GS}} = \hat{r}^0_{\mathrm{GS}} + \hat{r}^X_{\mathrm{GS}} \qquad \hat{r}_{\mathrm{thr}} = \hat{r}^{\mathrm{P}}_{\mathrm{thr}} + \hat{r}^{\mathrm{NP}}_{\mathrm{thr}} \qquad \hat{r}_{\mathrm{VY}} = \hat{r}^U_{\mathrm{VY}} + \hat{r}^\Pi_{\mathrm{VY}} \tag{B.2}$$

$$\begin{aligned}
&\hat{r}_{\mathrm{pot}} = \mathrm{e}^{K/2}W \qquad \hat{r}^X_{\mathrm{GS}} = -\frac{\delta_X}{8}(\overline{\mathcal{D}}^2 - 8R)(LV_X)\\
&\hat{r}^{\mathrm{P}}_{\mathrm{thr}} = \frac{1}{4}\sum_I \sum_{(a)\notin G_C} (\mathcal{W}\mathcal{W})_{(a)} b^I_{(a)} \ln \eta^{-2}(T^I)\\
&\hat{r}^{\mathrm{NP}}_{\mathrm{thr}} = \frac{1}{4}\sum_I \sum_{(a)\in G_C} U_{(a)} b^I_{(a)} \ln \eta^{-2}(T^I)\\
&\hat{r}^U_{\mathrm{VY}} = \frac{1}{4}\sum_{(a)\in G_C} b'_{(a)} U_{(a)} \ln(\mathrm{e}^{-K/2}U_{(a)}/\mu^3)\\
&\hat{r}^\Pi_{\mathrm{VY}} = \frac{1}{4}\sum_{(a)\in G_C}\sum_\alpha b^\alpha_{(a)} U_{(a)} \ln\left(A^\alpha_{(a)}(\Phi)\Pi^\alpha\right)
\end{aligned} \tag{B.3}$$

$$\begin{aligned}
&\hat{r}_{\mathrm{kin}} = -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)[-2 + f(L)]\\
&\hat{r}^0_{\mathrm{GS}} = -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)[LS(\Phi, \overline{\Phi})].
\end{aligned} \tag{B.4}$$

It is convenient to introduce two holomorphic functionals $h^{(a)}(\Phi, \Pi)$ and $\hat{h}^{(a)}(\Phi)$ by the identifications

$$\hat{r}^\Pi_{\mathrm{VY}} + \hat{r}^{\mathrm{NP}}_{\mathrm{thr}} \equiv \frac{1}{4}\sum_{(a)\in G_C} U_{(a)} h^{(a)}(\Phi, \Pi) \qquad \hat{r}^{\mathrm{P}}_{\mathrm{thr}} \equiv \frac{1}{4}\sum_{(a)\notin G_C} (\mathcal{W}\mathcal{W})_{(a)} \hat{h}^{(a)}(\Phi, \Pi). \tag{B.5}$$

From expressions (B.3) we see that

$$h^{(a)} = \sum_\alpha b^\alpha_{(a)} \ln\left(A^\alpha_{(a)}(\Phi)\Pi^\alpha\right) + \hat{h}^{(a)} \qquad \hat{h}^{(a)} = \sum_I b^I_{(a)} \ln \eta^{-2}(T^I). \tag{B.6}$$

It is worth noting that since our component expansions are written in terms of $h^{(a)}, \hat{h}^{(a)}$, our results are more general than (B.6), and accommodate any assumptions of the form (B.5).

For any of the $\hat{r}_i$ defined above, we have from (BGG-4.4.22) that the corresponding component Lagrangian is given in terms of the $\theta = \bar{\theta} = 0$ limit of spinorial derivatives:

$$\mathcal{L}_i = e\left[-\frac{1}{4}\mathcal{D}^2\hat{r}_i|_{(0,0)} + \frac{\mathrm{i}}{2}(\bar{\psi}_m\bar{\sigma}^m)^\alpha\mathcal{D}_\alpha\hat{r}_i|_{(0,0)} - (\overline{M} + \bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n)\hat{r}_i|_{(0,0)}\right] + \mathrm{h.c.} \tag{B.7}$$

Here, e is the determinant of the ordinary vierbein; i.e., the usual $\sqrt{-\det g}$ factor. In each case $\hat{r}_i$ has $U(1)_K$ weight 2. Let the symbol $\mathcal{D}$ denote covariant differentiation including $U(1)_K$ and D covariant differentiation not including $U(1)_K$. Then

$$\mathcal{D}_\alpha \hat{r}_i = (D_\alpha + 2A_\alpha)\hat{r}_i \qquad \mathcal{D}^2\hat{r}_i = (D^\alpha + A^\alpha)\,\mathcal{D}_\alpha \hat{r}_i \tag{B.8}$$

where the superform A is the $U(1)_K$ connection[10].

Appendix C. Geometric relations

Here we briefly discuss methods based on the $U(1)_K$ superspace geometry that are used in the more difficult expansions. The first set arises in the kinetic part of the Lagrangian $\mathcal{L}_{\text{kin}}$, defined in (2.20). The second set occurs in the Green–Schwarz (GS) counterterm Lagrangian $\mathcal{L}^0_{\text{GS}}$, associated with duality group invariance, defined in (2.25).

The difficulty that is encountered in evaluating $\mathcal{L}_{\text{kin}}$ is the computation of the component field expansion of

$$\mathcal{D}_\alpha R|_{(0,0)} \qquad \overline{\mathcal{D}}^{\dot\alpha}\bar{R}|_{(0,0)} \qquad \mathcal{D}^2 R|_{(0,0)} + \text{h.c.} \tag{C.1}$$

Techniques for the evaluation of these in the presence of a linear multiplet were developed in [16]. However in that case the L-dependent Kähler potential is $k(L) = \alpha \ln L$ and some simplifications occur; furthermore, our conventions for the component field definitions differ slightly; thus in the present context we must recalculate these expansions. We now detail the techniques and arrange the expressions that are evaluated in appendices E and F.

For the evaluation of $\mathcal{D}_\alpha R|_{(0,0)}$, we appeal to the identity (BGG-3.4.42),

$$\mathcal{D}_\alpha R = -\tfrac{1}{3}X_\alpha - \tfrac{2}{3}(\sigma^{cb}\epsilon)_{\alpha\gamma}T_{cb}{}^{\gamma} \tag{C.2}$$

where X_α is the chiral field strength associated with the Kähler potential:

$$X_\alpha \equiv -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K. \tag{C.3}$$

As originally described in [16], the field strength X_α contains $\mathcal{D}_\alpha R$ because of the L-dependence in K (cf equation (2.18) and the modified linearity conditions (A.1). Thus we extract the $\mathcal{D}_\alpha R$ contained in X_α so that we can solve for it explicitly, following the methods of [16]—reviewed in BGG section 5.4. This involves the definitions[11].

$$\begin{aligned}
X_\alpha &= X_{0\alpha} + Z_\alpha - Lk'(L)\mathcal{D}_\alpha R \\
X_{0\alpha} &\equiv -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha G(\Phi, \overline{\Phi}) \\
Z_\alpha &\equiv Lk'(L)\mathcal{D}_\alpha R - \tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha k(L).
\end{aligned} \tag{C.4}$$

Using (C.4) we rewrite (C.2) as

$$(k'L - 3)\mathcal{D}_\alpha R = X_{0\alpha} + Z_\alpha + 2(\sigma^{cb}\epsilon)_{\alpha\gamma}T_{cb}{}^{\gamma} \tag{C.5}$$

where now, as it turns out, $\mathcal{D}_\alpha R$ will not appear on the right-hand side when we work out the component expansion. Equation (C.5) is in agreement with (BGG-5.4.6). After considerable manipulation we find for $\mathcal{D}_\alpha R|_{(0,0)}$ the expansion given in (E.3) and (E.4). One obtains $\overline{\mathcal{D}}^{\dot\alpha}\overline{R}|_{(0,0)}$ by Hermitian conjugation.

For the evaluation of $(\mathcal{D}^2 R + \text{h.c.})|_{(0,0)}$ we appeal to (BGG-3.4.44):

$$\mathcal{D}^2 R + \text{h.c.} = -\tfrac{2}{3}R_{ba}{}^{ba} - \tfrac{1}{3}(\mathcal{D}^\alpha X_\alpha + \text{h.c.}) + 4G^a G_a + 32R\overline{R}. \tag{C.6}$$

Similar to the situation described in the previous paragraph, we need to extract $(\mathcal{D}^2 R + \text{h.c.})$

[10] The quantity A_m used above is related via $A_m = (E_m{}^B A_B)|_{(0,0)}$, where B runs over $b, \beta, \dot\beta$.

[11] Note that the quantity Y_α appearing in (BGG-5.4.5) is related to the present notation by $Y_\alpha \equiv X_{0\alpha} + Z_\alpha$.

from $(\mathcal{D}^\alpha X_\alpha + \text{h.c.})$ due to the L-dependence in K. With definition (C.4) it is not hard to show that

$$(k'L-3)(\mathcal{D}^2 R+\text{h.c.}) = 2R_{ba}{}^{ba} - 12G^a G_a - 96R\overline{R} + (\mathcal{D}^\alpha X_{0\alpha} + \mathcal{D}^\alpha Z_\alpha + \text{h.c.})$$
$$- (k'+k''L)(\mathcal{D}^\alpha L \mathcal{D}_\alpha R + \text{h.c.}) \tag{C.7}$$

in agreement with (BGG-5.4.8). Taking the $\theta = \bar\theta = 0$ part of this expression yields

$$(k'\ell-3)(\mathcal{D}^2 R+\text{h.c.})|_{(0,0)} = 2\mathcal{R} - \tfrac{4}{3}b^m b_m - \tfrac{8}{3}M\overline{M} + (\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)} + \mathcal{D}^\alpha Z_\alpha|_{(0,0)} + \text{h.c.})$$
$$- (k'+k''\ell)(\varphi^\alpha \mathcal{D}_\alpha R|_{(0,0)} + \text{h.c.}) \tag{C.8}$$

where $\mathcal{R}$ is the spacetime Ricci scalar. Note that in the special case $k(\ell) = \alpha \ln \ell$ considered in [16] we have $k'+k''\ell = 0$. This eliminates the last term and simplifies many of the expressions given below.

The component field expansion of $(\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)} + \text{h.c.})$ is obtained from (BGG-4.2.13) provided we take $K(\Phi,\overline{\Phi}) \to G(\Phi,\overline{\Phi})$ in their expressions for derivatives of the Kähler potential. This leaves only $(\mathcal{D}^\alpha Z_\alpha|_{(0,0)} + \text{h.c.})$ to be determined; we have evaluated this in (E.7).

For the evaluation of $\mathcal{L}^0_{\text{GS}}$ it was shown in [16] how to proceed through the chiral field strength for the GS counterterm potential $S(\Phi,\overline{\Phi})$:

$$X_{S\alpha} = -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha S. \tag{C.9}$$

The evaluation of $X_{S\alpha}$ is more complicated than that of $X_{0\alpha} \equiv -\frac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha G$: whereas $G_{k\bar{k};\ell} \equiv 0$, the corresponding quantity $S_{k\bar{k};\ell}$ does not necessarily vanish. However, the organization of the calculation around this field strength proves productive and leads directly to the results of appendix F.

Appendix D. Expansion of $\mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{VY}} + \mathcal{L}_{\text{thr}} + \mathcal{L}^X_{\text{GS}}$

Here the expansions are straightforward. See appendix B for superfield definitions of various parts of the Lagrangian given here:

$$\frac{1}{e}\mathcal{L}_{\text{pot}} = \mathrm{e}^{K/2}\left\{-\frac{1}{2}(W_{k;\ell} + WG_{k;\ell} + W_\ell G_k + W_k G_\ell + WG_k G_\ell)(\chi^k\chi^\ell)\right.$$
$$-\frac{1}{4}W(k''+k'^2)(\varphi\varphi) - [\overline{M} + (\bar\psi_m\bar\sigma^{mn}\bar\psi_n)]W$$
$$+(W_k+WG_k)\left[F^k - \frac{1}{\sqrt2}k'(\varphi\chi^k) + \frac{\mathrm{i}}{\sqrt2}(\bar\psi^m\bar\sigma_m\chi^k)\right]$$
$$\left.+Wk'\left[\frac14\bar u - \frac14\mathrm{Tr}(\bar\lambda\bar\lambda) + \frac13\overline{M}\ell + \frac{\mathrm{i}}{2}(\bar\psi^m\bar\sigma_m\varphi)\right]\right\} + \text{h.c.} \tag{D.1}$$

$$\frac{1}{e}\mathcal{L}^U_{\text{VY}} = \sum_{(a)\in G_C}\frac{b'_{(a)}}{4}\left\{-\frac{1}{2u_{(a)}}(\Lambda\Lambda)_{(a)} + \frac14 k''(\varphi\varphi) + \frac12 G_{k;\ell}(\chi^k\chi^\ell) + \frac{1}{\sqrt2}k'(\Lambda_{(a)}\varphi)\right.$$
$$-[\overline{M} + (\bar\psi_m\bar\sigma^{mn}\bar\psi_n)]u_{(a)}\ln(\mathrm{e}^{-K/2}u_{(a)}/\mu^3)$$
$$-k'u_{(a)}\left[\frac14\bar u - \frac14\mathrm{Tr}(\bar\lambda\bar\lambda) + \frac13\overline{M}\ell + \frac{\mathrm{i}}{2}(\bar\psi^m\bar\sigma_m\varphi)\right]$$
$$-G_k\left[u_{(a)}F^k + \frac{\mathrm{i}}{\sqrt2}u_{(a)}(\bar\psi^m\bar\sigma_m\chi^k) - (\Lambda_{(a)}\chi^k)\right]$$
$$\left.+\ln\left(\mathrm{e}^{1-\frac{K}{2}}u_{(a)}/\mu^3\right)\left[F_{U_{(a)}} + \frac{\mathrm{i}}{\sqrt2}(\bar\psi^m\bar\sigma_m\Lambda_{(a)})\right]\right\} + \text{h.c.} \tag{D.2}$$

$$\frac{1}{e}\mathcal{L}_{\mathrm{VY}}^{\Pi}+\frac{1}{e}\mathcal{L}_{\mathrm{thr}}^{\mathrm{NP}}=\sum_{(a)\in G_C}\frac{1}{4}\left\{h^{(a)}\left[F_{U_{(a)}}+\frac{\mathrm{i}}{\sqrt{2}}(\bar{\psi}^m\bar{\sigma}_m\Lambda_{(a)})-u_{(a)}(\overline{M}+(\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n))\right]\right.$$
$$\left.+h_k^{(a)}\left[u_{(a)}F^k+\frac{\mathrm{i}}{\sqrt{2}}u_{(a)}(\bar{\psi}^m\bar{\sigma}_m\chi^k)-(\Lambda_{(a)}\chi^k)\right]-\frac{1}{2}h_{k;\ell}^{(a)}u_{(a)}(\chi^k\chi^\ell)\right\}+\mathrm{h.c.} \tag{D.3}$$

$$\frac{1}{e}\mathcal{L}_{\mathrm{thr}}^{\mathrm{P}}=\sum_{(a)\notin G_C}\frac{1}{4}\left\{\hat{h}^{(a)}\left[(\lambda\lambda)_{(a)}(\overline{M}+(\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n))-\frac{1}{2}f_{(a)}^{mn}f_{(a)mn}\right.\right.$$
$$-\frac{\mathrm{i}}{4}\epsilon^{mnpq}f_{(a)mn}f_{(a)pq}-2\mathrm{i}(\lambda\sigma^m\mathcal{D}_m\bar{\lambda})_{(a)}+\mathbf{D}_{(a)}^2-\overline{M}(\lambda\lambda)_{(a)}$$
$$+((\psi_m\sigma^{pq}\sigma^m\bar{\lambda}_{(a)})+(\bar{\psi}_m\bar{\sigma}^{pq}\bar{\sigma}^m\lambda_{(a)}))$$
$$\times\left(\mathrm{i}f_{(a)pq}+\frac{1}{2}(\psi_p\sigma_q\bar{\lambda}_{(a)})+\frac{1}{2}(\bar{\psi}_p\bar{\sigma}_q\lambda_{(a)})\right)$$
$$\left.-\frac{\mathrm{i}}{2}\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\lambda_{(a)})((\psi_p\sigma_q\bar{\lambda}_{(a)})+(\bar{\psi}_p\bar{\sigma}_q\lambda_{(a)}))\right]$$
$$-\hat{h}_k^{(a)}\left[(\lambda\lambda)_{(a)}\left(F^k+\frac{\mathrm{i}}{\sqrt{2}}(\bar{\psi}^m\bar{\sigma}_m\chi^k)\right)-\mathrm{i}\sqrt{2}(\chi^k\lambda_{(a)})\mathbf{D}_{(a)}\right.$$
$$\left.\left.+\sqrt{2}(\chi^k\sigma^{mn}\lambda^{(a)})\hat{f}_{(a)mn}\right]-\frac{1}{2}\hat{h}_{k;\ell}^{(a)}u_{(a)}(\chi^k\chi^\ell)\right\}+\mathrm{h.c.} \tag{D.4}$$

$$\frac{1}{e}\mathcal{L}_{\mathrm{GS}}^{X}=\frac{\delta_X}{4}a_{Xm}\{2B^m+\mathrm{Tr}(\bar{\lambda}\bar{\sigma}^m\lambda)+2\mathrm{i}[(\varphi\sigma^{nm}\psi_n)+(\bar{\psi}_n\bar{\sigma}^{nm}\bar{\varphi})]$$
$$-2\ell(\bar{\psi}^n\bar{\sigma}^m\bar{\psi}_n)+\mathrm{i}\ell\epsilon^{mnpq}(\bar{\psi}_n\bar{\sigma}_p\psi_q)\}+\frac{\delta_X}{4}\{\mathrm{i}[(\varphi\lambda_X)-(\bar{\varphi}\bar{\lambda}_X)]$$
$$+\ell[(\bar{\psi}^m\bar{\sigma}_m\lambda_X)+(\bar{\lambda}_X\bar{\sigma}^m\psi_m)]+2\ell\mathbf{D}_X\}. \tag{D.5}$$

Here we have evaluated $\mathcal{L}_{\mathrm{GS}}^{X}$ in Wess–Zumino (WZ) gauge (see equation (A.12)).

Appendix E. Expansion of $\mathcal{L}_{\mathrm{kin}}$

The kinetic Lagrangian $\mathcal{L}_{\mathrm{kin}}$, defined in (2.20), is obtained from the component expansion of $\hat{r}_{\mathrm{kin}}$, defined in (B.4). With some effort the following expressions are obtained:

$$\hat{r}_{\mathrm{kin}}|_{(0,0)}=-\tfrac{1}{8}f''(\bar{\varphi}\bar{\varphi})+\tfrac{1}{8}f'\big(u-(\lambda^{(a)}\lambda_{(a)})+\tfrac{4}{3}M\ell\big)+\tfrac{1}{6}(2-f)M \tag{E.1}$$

$$\mathcal{D}_\alpha\hat{r}_{\mathrm{kin}}|_{(0,0)}=-\frac{1}{8}f'''\varphi_\alpha(\bar{\varphi}\bar{\varphi})+\frac{1}{8}f''\varphi_\alpha\left(u-(\lambda^{(a)}\lambda_{(a)})+\frac{4}{3}M\ell\right)$$
$$+\frac{1}{4\sqrt{2}}f'\Lambda_\alpha-\frac{\mathrm{i}}{4}f'\lambda_\alpha^{(a)}\mathbf{D}_{(a)}+\frac{1}{4}f'(\sigma^{nm}\lambda^{(a)})_\alpha\hat{f}_{(a)nm}$$
$$-\frac{1}{4}f''(\sigma^m\bar{\varphi})_\alpha B_m+\frac{\mathrm{i}}{4}f''(\sigma^m\bar{\varphi})_\alpha\Delta_m\ell+\frac{1}{6}f''\ell(\sigma^m\bar{\varphi})_\alpha b_m$$
$$+\frac{1}{4}f''\lambda_\alpha^{(a)}(\bar{\lambda}_{(a)}\bar{\varphi})+X_{0\alpha}|_{(0,0)}+Z_\alpha|_{(0,0)}+2(\sigma^{cb})_\alpha{}^\gamma T_{cb\gamma}|_{(0,0)}. \tag{E.2}$$

We have left abbreviated $T_{cb\gamma}|_{(0,0)}$, which is given above in (A.19), as well as

$$\begin{aligned}
Z_\alpha|_{(0,0)} = &-\frac{1}{8}k'''\varphi_\alpha(\bar\varphi\bar\varphi) - \frac{\mathrm{i}}{4}k''(\sigma^m\bar\varphi)_\alpha\Delta_m\ell - \frac{\mathrm{i}}{4}k'\lambda^{(a)}_\alpha\mathbf{D}_{(a)}\\
&+\frac{1}{8}(k''\varphi_\alpha + \mathrm{i}k'(\sigma^m\bar\psi_m)_\alpha)\left(u - (\lambda^{(a)}\lambda_{(a)}) + \frac{4}{3}M\ell\right)\\
&+\frac{1}{6}k''\ell(\sigma^m\bar\varphi)_\alpha b_m - \frac{1}{4}k''(\sigma^m\bar\varphi)_\alpha B_m - \frac{1}{4}k''\lambda^{(a)}_\alpha(\bar\lambda_{(a)}\bar\varphi)\\
&+\frac{1}{4}k'(\sigma^{nm}\lambda^{(a)})_\alpha\hat f_{(a)nm} + \frac{1}{4\sqrt{2}}k'\Lambda_\alpha + \frac{1}{6}k'M\varphi_\alpha + \frac{1}{6}k'(\sigma^m\bar\varphi)_\alpha b_m\\
&-\frac{\mathrm{i}}{2}k'(\sigma^m\mathcal{D}_m\bar\varphi)_\alpha + \frac{\mathrm{i}}{4}k'(\sigma^m\bar\sigma^n\psi_m)_\alpha\left(\mathrm{i}\Delta_n\ell + \frac{2}{3}\ell b_n - B_n\right)\\
&+\frac{\mathrm{i}}{4}k'(\sigma^m\bar\lambda^{(a)})_\alpha(\psi_m\lambda_{(a)})
\end{aligned} \tag{E.3}$$

$$X_{0\alpha}|_{(0,0)} = \frac{1}{\sqrt{2}}\overline{F}^{\bar k}G_{k\bar k}\chi^k_\alpha - \frac{\mathrm{i}}{\sqrt{2}}G_{k\bar k}(\sigma^m\bar\chi^{\bar k})_\alpha\Delta_m\phi^k + \mathrm{i}G_k\lambda^{(a)}_\alpha(T_{(a)}\phi)^k. \tag{E.4}$$

These quantities also appear in the final piece that contributes to $\mathcal{L}_{\mathrm{kin}}$:

$$\begin{aligned}
(\mathcal{D}^2\hat r_{\mathrm{kin}} + \mathrm{h.c.})|_{(0,0)} = &-\frac{1}{4}f''''(\varphi\varphi)(\bar\varphi\bar\varphi) + \frac{2}{3}(f'' + \ell f''')(\varphi\sigma^m\bar\varphi)b_m\\
&+\frac{1}{4}f'''\left[(\bar\varphi\bar\varphi)\left(\overline{u} - (\bar\lambda^{(a)}\bar\lambda_{(a)}) + \frac{4}{3}\overline{M}\ell\right) + \mathrm{h.c.}\right]\\
&- f'''(\varphi\sigma^m\bar\varphi)B_m + f'''(\varphi\lambda^{(a)})(\bar\varphi\bar\lambda_{(a)}) + \frac{5}{6}f''\left[M(\varphi\varphi) + \mathrm{h.c.}\right]\\
&+ f''\left[\mathrm{i}(\mathcal{D}_m\varphi\sigma^m\bar\varphi) - \frac{\mathrm{i}}{2}(\lambda^{(a)}\sigma^m\bar\varphi)(\bar\lambda_{(a)}\bar\psi_m)\right.\\
&+\frac{\mathrm{i}}{2}(\bar\psi_m\bar\sigma^n\sigma^m\bar\varphi)\left(\mathrm{i}\Delta_n\ell + B_n - \frac{2}{3}\ell b_n\right)\\
&\left.-\frac{\mathrm{i}}{4}(\psi_m\sigma^m\bar\varphi)\left(\overline{u} - (\bar\lambda^{(a)}\bar\lambda_{(a)}) + \frac{4}{3}\overline{M}\ell\right) + \mathrm{h.c.}\right]\\
&+\frac{1}{\sqrt{2}}f''[(\varphi\Lambda) + \mathrm{h.c.}] - f''\Delta^m\ell\Delta_m\ell\\
&+ f''\left(B^m - \frac{2}{3}\ell b^m\right)\left(B_m - \frac{2}{3}\ell b_m\right) - \frac{1}{2}f'(F_U + \overline{F}_{\overline{U}})\\
&-\frac{1}{2}f''(\lambda^{(a)}\lambda^{(b)})(\bar\lambda_{(a)}\bar\lambda_{(b)}) + f''\left(B_m - \frac{2}{3}\ell b_m\right)\mathrm{Tr}(\lambda\sigma^m\bar\lambda)\\
&-\frac{1}{4}f''\left(u - (\lambda^{(a)}\lambda_{(a)}) + \frac{4}{3}M\ell\right)\left(\bar u - (\bar\lambda^{(a)}\bar\lambda_{(a)}) + \frac{4}{3}\overline{M}\ell\right)\\
&+ f''[-\mathrm{i}(\varphi\lambda^{(a)})\mathbf{D}_{(a)} + (\varphi\sigma^{nm}\lambda^{(a)})\hat f_{(a)nm} + \mathrm{h.c.}] + 2\mathcal{R} - \frac{4}{3}b^m b_m - \frac{8}{3}M\overline{M}\\
&+\left(\frac{k' + \ell k''}{1 - \frac{1}{3}\ell k'}\right)\left[\varphi^\alpha X_{0\alpha}|_{(0,0)} + \varphi^\alpha Z_\alpha|_{(0,0)} + 2(\varphi\sigma^{cb})_\alpha{}^\gamma T_{cb\gamma}|_{(0,0)} + \mathrm{h.c.}\right]\\
&+[\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)} + \mathcal{D}^\alpha Z_\alpha|_{(0,0)} + \mathrm{h.c.}] + \frac{1}{8}f'[\mathcal{D}^2\,\mathrm{Tr}(\mathcal{W}\mathcal{W})|_{(0,0)} + \mathrm{h.c.}].
\end{aligned} \tag{E.5}$$

Here we have abbreviated $\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)}$ and $\mathcal{D}^\alpha Z_\alpha|_{(0,0)}$. The quantity $\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)}$ is obtained from (BGG-4.2.13) with the replacement $K(\phi,\bar\phi)\to G(\phi,\bar\phi)$. Thus,

$$\begin{aligned}
\mathcal{D}^\alpha X_{0\alpha}|_{(0,0)} = G_{k\bar k}\Bigg[& 2\mathcal{D}^m\phi^k\mathcal{D}_m\bar\phi^{\bar k} - 2F^k\overline{F}^{\bar k} - (\psi^m\chi^k)(\bar\psi_m\bar\chi^{\bar k}) \\
&+ \mathrm{i}(\chi^k\sigma^m\mathcal{D}_m\bar\chi^{\bar k}) - \mathrm{i}(\mathcal{D}_m\chi^k\sigma^m\bar\chi^{\bar k}) \\
&+ \mathrm{i}2\sqrt{2}(\chi^k\lambda^{(a)})(\bar\phi T_{(a)})^{\bar k} - \mathrm{i}2\sqrt{2}(\bar\chi^{\bar k}\bar\lambda^{(a)})(T_{(a)}\phi)^k \\
&+ \frac{\mathrm{i}}{\sqrt{2}}\overline{F}^{\bar k}(\bar\psi_m\bar\sigma^m\chi^k) + \frac{\mathrm{i}}{\sqrt{2}}F^k(\psi_m\sigma^m\bar\chi^{\bar k}) \\
&+ \frac{1}{\sqrt{2}}((\bar\psi_m\bar\sigma^n\sigma^m\bar\chi^{\bar k}) - 2(\bar\psi^n\bar\chi^{\bar k}))\Delta_n\phi^k \\
&+ \frac{1}{\sqrt{2}}((\psi_m\sigma^n\bar\sigma^m\chi^k) - 2(\psi^n\chi^k))\Delta_n\bar\phi^{\bar k}\Bigg] \\
&- \frac{1}{2}R_{k\bar k\ell\bar\ell}(\chi^k\chi^\ell)(\bar\chi^{\bar k}\bar\chi^{\bar\ell}) + 2G_k\mathbf{D}^{(a)}(T_{(a)}\phi)^k.
\end{aligned} \tag{E.6}$$

After no small effort we obtain

$$\begin{aligned}
(\mathcal{D}^\alpha Z_\alpha + \text{h.c.})|_{(0,0)} = &-\frac14 k''''(\varphi\varphi)(\bar\varphi\bar\varphi) - \frac12 k'\,(F_U + \text{h.c.}) + k''\Delta^m\ell\Delta_m\ell \\
&+ 2k'\mathcal{D}^m\Delta_m\ell + \frac{1}{2\sqrt{2}}k''[(\Lambda\varphi)+\text{h.c.}] + \frac12 k''\mathbf{D}^{(a)}[\mathrm{i}(\varphi\lambda_{(a)}) + \text{h.c.}] \\
&+ \frac12 k''\hat f^{(a)}_{nm}[(\varphi\sigma^{nm}\lambda_{(a)}) + \text{h.c.}] - \frac12(k''+k''')(\varphi\lambda^{(a)})(\bar\varphi\bar\lambda_{(a)}) \\
&- \frac12 k''(\lambda^{(a)}\lambda^{(b)})(\bar\lambda_{(a)}\bar\lambda_{(b)}) - \left[k''b_m + \frac12(k''+k''')B_m\right](\varphi\sigma^m\bar\varphi) \\
&+ \left[\frac13(k'+2k''\ell)b_m - k''B_m\right](\lambda^{(a)}\sigma^m\bar\lambda_{(a)}) \\
&+ \left(\frac23\ell b^m - B^m\right)\left[\frac43 k'b_m + k''\left(\frac23\ell b^m - B^m\right)\right] \\
&- \frac14 k''\left(u - (\lambda^{(a)}\lambda_{(a)}) + \frac43 M\ell\right)\left(\bar u - (\bar\lambda^{(a)}\bar\lambda_{(a)}) + \frac43\overline{M}\ell\right) \\
&+ \Bigg\{\left(\bar u - (\bar\lambda^{(a)}\bar\lambda_{(a)}) + \frac43\overline{M}\ell\right)\Bigg[-\frac16 k'M + \frac14 k'''(\bar\varphi\bar\varphi) \\
&- \frac{\mathrm{i}}{8}k''(\psi_m\sigma^m\bar\varphi) + \frac14 k'(\psi^m\psi_m)\Bigg] + \text{h.c.}\Bigg\} \\
&+ \frac13 k''[M(\varphi\varphi) + \text{h.c.}] + \frac12 k''[\mathrm{i}(\mathcal{D}_m\varphi\sigma^m\bar\varphi) + \text{h.c.}] \\
&+ \frac14 k''\left[\mathrm{i}(\bar\psi_m\bar\sigma^n\sigma^m\bar\varphi)\left(\mathrm{i}\Delta_n\ell + B_n - \frac23\ell b_n\right) + \text{h.c.}\right] \\
&- \frac14 k''[(\lambda^{(a)}\sigma^m\bar\varphi)(\bar\lambda_{(a)}\bar\psi_m) + \text{h.c.}] \\
&+ k'(\psi^m\lambda^{(a)})(\bar\psi_m\bar\lambda_{(a)}) - k'[(\psi^m\mathcal{D}_m\varphi) + \text{h.c.}] \\
&- \frac16 k'b_n[\mathrm{i}(\psi_m\sigma^n\bar\sigma^m\varphi) + \text{h.c.}] + \frac16 k'[\mathrm{i}\overline{M}(\psi_m\sigma^m\bar\varphi) + \text{h.c.}] \\
&+ k'(\psi^m\sigma^n\bar\psi_m)\left(B_n - \frac23\ell b_n\right) - \frac13 k'b_m(\bar\lambda^{(a)}\bar\sigma^m\lambda_{(a)})
\end{aligned}$$

$$+\left(\frac{k'+3k''\ell}{k'\ell-3}\right)[\varphi^{\alpha}X_{0\alpha}|_{(0,0)}+\varphi^{\alpha}Z_{\alpha}|_{(0,0)}+\text{h.c.}]$$
$$+\left(\frac{k'(4-k'\ell+k''\ell^2)}{k'\ell-3}\right)[(\varphi\sigma^{cb})^{\alpha}T_{cb\alpha}|_{(0,0)}+\text{h.c.}]$$
$$+\frac{1}{8}k'[\mathcal{D}^2\,\text{Tr}(\mathcal{W}\mathcal{W})|_{(0,0)}+\text{h.c.}]. \tag{E.7}$$

Together with the notation defined in appendix A, equations (E.1)–(E.7) provide the full component expansion of $\mathcal{L}_{\text{kin}}$.

Appendix F. Expansion of $\mathcal{L}^{0}_{\text{GS}}$

A tedious calculation, using the methods described in appendix C, yields

$$\hat{r}^{0}_{\text{GS}}\big|_{(0,0)}=\frac{1}{8}S\,[u-\text{Tr}(\lambda\lambda)]+\frac{1}{2}S_{\bar{k}}\left[\ell\overline{F}^{\bar{k}}-\frac{1}{\sqrt{2}}(\bar{\varphi}\bar{\chi}^{\bar{k}})\right]-\frac{1}{4}\ell S_{\bar{k};\bar{\ell}}(\bar{\chi}^{\bar{k}}\bar{\chi}^{\bar{\ell}}) \tag{F.1}$$

$$\begin{aligned}\mathcal{D}_{\alpha}\hat{r}^{0}_{\text{GS}}\big|_{(0,0)}=&\frac{1}{4}S\left[\frac{1}{\sqrt{2}}\Lambda_{\alpha}-\text{i}\lambda^{(a)}_{\alpha}\mathbf{D}_{(a)}+(\sigma^{nm}\lambda^{(a)})_{\alpha}\hat{f}_{(a)nm}\right]\\&+S_k\left[\frac{1}{4\sqrt{2}}\chi^{k}_{\alpha}(u-\text{Tr}(\lambda\lambda))+\text{i}\ell\lambda^{(a)}_{\alpha}(T_{(a)}\phi)^{k}\right]\\&+\frac{1}{2}S_{\bar{k}}\left[\overline{F}^{\bar{k}}(\varphi_{\alpha}-\text{i}\ell(\sigma^{m}\bar{\psi}_{m})_{\alpha})-\frac{1}{\sqrt{2}}\lambda^{(a)}_{\alpha}(\bar{\lambda}_{(a)}\bar{\chi}^{\bar{k}})\right.\\&+\Delta_m\bar{\phi}^{\bar{k}}\left(\text{i}(\sigma^{m}\bar{\varphi})_{\alpha}+\ell(\sigma^{n}\bar{\sigma}^{m}\psi_{n})_{\alpha}\right)\\&\left.+\sqrt{2}\text{i}\ell(\sigma^{m}\mathcal{D}_m\bar{\chi}^{\bar{\ell}})_{\alpha}+\frac{1}{\sqrt{2}}(\sigma^{m}\bar{\chi}^{\bar{k}})_{\alpha}(\text{i}\Delta_m\ell-B^{m})\right]\\&+\frac{1}{\sqrt{2}}S_{k\bar{k}}\chi^{k}_{\alpha}\left[\ell\overline{F}^{\bar{k}}-\frac{1}{\sqrt{2}}(\bar{\varphi}\bar{\chi}^{\bar{k}})\right]-\frac{1}{2\sqrt{2}}\ell S_{k\bar{k};\bar{\ell}}\chi^{k}_{\alpha}(\bar{\chi}^{\bar{k}}\bar{\chi}^{\bar{\ell}})\\&+S_{\bar{k};\bar{\ell}}\left[\frac{\text{i}}{\sqrt{2}}\ell\Delta_m\bar{\phi}^{\bar{k}}(\sigma^{m}\bar{\chi}^{\bar{\ell}})_{\alpha}-\frac{1}{4}\varphi_{\alpha}(\bar{\chi}^{\bar{k}}\bar{\chi}^{\bar{\ell}})\right]\end{aligned} \tag{F.2}$$

$$\begin{aligned}\mathcal{D}^2\hat{r}^{0}_{\text{GS}}\big|_{(0,0)}=&-\frac{1}{2}SF_U+S_k\left[-\frac{1}{2}F^{k}(u-(\lambda^{(a)}\lambda_{(a)}))+\frac{1}{2}(\chi^{k}\Lambda)\right.\\&+\text{i}(\varphi\lambda^{(a)})(T_{(a)}\phi)^{k}+\mathbf{D}^{(a)}\left(2\ell(T_{(a)}\phi)^{k}-\frac{\text{i}}{\sqrt{2}}(\chi^{k}\lambda^{(a)})\right)\\&\left.+\sqrt{2}\text{i}\ell(\lambda^{(a)}T_{(a)}\chi)^{k}+\frac{1}{\sqrt{2}}(\chi^{k}\sigma^{nm}\lambda^{(a)})\hat{f}_{(a)nm}\right]\\&+S_{\bar{k}}\left\{-\frac{1}{2}\left(\overline{F}^{\bar{k}}+\frac{\text{i}}{\sqrt{2}}(\psi_m\sigma^{m}\bar{\chi}^{\bar{k}})\right)\left(\bar{u}-(\bar{\lambda}^{(a)}\bar{\lambda}_{(a)})+\frac{4}{3}\overline{M}\ell\right)\right.\\&+2\text{i}\Delta_m\bar{\phi}^{\bar{k}}(\text{i}\Delta_m\ell-B_m)+\sqrt{2}\text{i}\mathcal{D}_m(\varphi\sigma^{m}\bar{\chi}^{\bar{k}})\\&-\text{i}\overline{F}^{\bar{k}}(\varphi\sigma^{m}\bar{\psi}_m)+\Delta_n\bar{\phi}^{\bar{k}}(\varphi\sigma^{m}\bar{\sigma}^{n}\psi_m)+\sqrt{2}\overline{M}(\bar{\varphi}\bar{\chi}^{\bar{k}})\\&+\frac{1}{2}(\bar{\Lambda}\bar{\chi}^{\bar{k}})-\frac{\text{i}}{\sqrt{2}}(\lambda^{(a)}\sigma^{m}\bar{\chi}^{\bar{k}})(\bar{\lambda}_{(a)}\bar{\psi}_m)+\frac{\text{i}}{\sqrt{2}}\mathbf{D}^{(a)}(\bar{\lambda}_{(a)}\bar{\chi}^{\bar{k}})\\&+\frac{\text{i}}{\sqrt{2}}(\bar{\psi}_m\bar{\sigma}^{n}\sigma^{m}\bar{\chi}^{\bar{k}})(\text{i}\Delta_n\ell+B_n-\ell b_n)\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\sqrt{2}}(\bar\lambda^{(a)}\bar\sigma^{nm}\bar\chi^{\bar k})\hat f_{(a)nm}+\mathrm{i}\Delta_m\bar\phi^{\bar k}(\lambda^{(a)}\sigma^m\bar\lambda_{(a)})\\
&+\mathrm{i}(\bar\phi T_{(a)})^{\bar k}[(\varphi\lambda^{(a)})-2(\bar\lambda^{(a)}\bar\varphi)]-2\ell\mathcal{D}^m\mathcal{D}_m\bar\phi^k\\
&-\frac{\mathrm{i}}{3\sqrt{2}}\ell\overline{M}(\psi_m\sigma^m\bar\chi^{\bar k})+\sqrt{2}[\mathcal{D}^m(\bar\psi_m\bar\chi^{\bar k})+(\bar\psi^m\mathcal{D}_m\bar\chi^{\bar k})]\\
&-\ell(\bar\phi T_{(a)})^{\bar k}[(\psi_m\sigma^m\bar\lambda^{(a)})-(\bar\psi_m\bar\sigma^m\lambda^{(a)})]\\
&+\mathrm{i}\ell\Delta_n\bar\phi^{\bar k}(\psi_m\sigma^n\bar\psi^m)-\ell\overline{F}^{\bar k}(\bar\psi^m\bar\psi_m)-\frac{4}{3}\ell\overline{MF}^{\bar k}\\
&+2\sqrt{2}\mathrm{i}\ell(\bar\chi T_{(a)}\bar\lambda^{(a)})^{\bar k}+2\sqrt{2}\mathrm{i}\ell\Gamma^{\bar k}_{\bar\ell\bar m}(\bar\phi T_{(a)})^{\bar m}(\bar\lambda^{(a)}\bar\chi^{\bar\ell})\\
&-R^{\bar k}{}_{\bar m j\bar\ell}\left[\frac{1}{2\sqrt{2}}(\varphi\chi^j)(\bar\chi^{\bar m}\bar\chi^{\bar\ell})+\mathrm{i}\ell\Delta_n\bar\phi^{\bar m}(\chi^j\sigma^n\bar\chi^{\bar\ell})\right]\Bigg\}\\
&+S_{k\bar k}\Bigg[\sqrt{2}\overline{F}^{\bar k}(\varphi\chi^k)+\sqrt{2}F^k(\bar\varphi\bar\chi^{\bar k})-2\ell F^k\overline{F}^{\bar k}\\
&-\sqrt{2}\mathrm{i}\ell(\chi^k\lambda^{(a)})(\bar\phi T_{(a)})^{\bar k}+\frac{1}{2}\ell R^{\bar k}{}_{\bar m p\bar\ell}(\chi^k\chi^p)(\bar\chi^{\bar m}\bar\chi^{\bar\ell})\\
&-\sqrt{2}\ell\Delta_n\phi^k(\bar\psi_m\bar\sigma^n\sigma^m\bar\chi^{\bar k})+(\chi^k\sigma^m\bar\chi^{\bar k})\left(\mathrm{i}\Delta_m\ell+\frac{2}{3}\ell b_m-B_m\right)\\
&-(\chi^k\lambda^{(a)})(\bar\chi^{\bar k}\bar\lambda_{(a)})+\sqrt{2}\mathrm{i}\Delta_m\bar\phi^{\bar k}(\chi^k\sigma^m\bar\varphi)\Bigg]\\
&+S_{k;\ell}\left[\frac{1}{4}(\chi^k\chi^\ell)(u-(\lambda^{(a)}\lambda_{(a)}))+\sqrt{2}\mathrm{i}\ell(\chi^\ell\lambda^{(a)})(T_{(a)}\phi)^k\right]\\
&+S_{\bar k;\bar\ell}\Bigg[\frac{1}{4}(\bar\chi^{\bar k}\bar\chi^{\bar\ell})\left(\overline{u}-(\bar\lambda^{(a)}\bar\lambda_{(a)})-\frac{4}{3}\overline{M}\ell\right)\\
&+\sqrt{2}\mathrm{i}\Delta_m\bar\phi^{\bar k}(\varphi\sigma^m\bar\chi^{\bar\ell})-2\ell\Delta_m\bar\phi^{\bar k}\Delta^m\bar\phi^{\bar\ell}\\
&+2\sqrt{2}\mathrm{i}\ell(\bar\lambda^{(a)}\bar\chi^{\bar k})(\bar\phi T_{(a)})^{\bar\ell}\Bigg]+S_{k\bar k;\bar\ell}\Bigg[-\frac{1}{\sqrt{2}}(\bar\chi^{\bar k}\bar\chi^{\bar\ell})(\varphi\chi^k)\\
&+\ell F^k(\bar\chi^{\bar k}\bar\chi^{\bar\ell})+2\mathrm{i}\ell\Delta_m\bar\phi^{\bar\ell}(\chi^k\sigma^m\bar\chi^{\bar k})\Bigg]\\
&+S_{k\bar\ell;\ell}\left[-\frac{1}{\sqrt{2}}(\chi^k\chi^\ell)(\bar\varphi\bar\chi^{\bar\ell})+\ell\overline{F}^{\bar\ell}(\chi^k\chi^\ell)\right]\\
&-\frac{1}{2}\ell S_{k\bar k;\ell;\bar\ell}(\chi^k\chi^\ell)(\bar\chi^{\bar k}\bar\chi^{\bar\ell})+\frac{1}{8}S\mathcal{D}^2\,\mathrm{Tr}(\mathcal{W}\mathcal{W})|_{(0,0)}\\
&+2\sqrt{2}\ell S_{\bar k}(\bar\chi^{\bar k}\bar\sigma^{cb})_{\dot\alpha}T_{cb}{}^{\dot\alpha}|_{(0,0)}.
\end{aligned}
\qquad\text{(F.3)}
$$

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